Sagdeev Potential Approach for Quantum Dust Ion Acoustic Waves in an Electron-Positron-Ion-Dusty plasma

Gadadhar Banerjee[1*] and Sarit Maitra[2]
[1] *Department of Basic Science and Humanities, University of Engineering and Management Kolkata, Kolkata-700160 and*
[2] *Department of Mathematics, National Institute of Technology Durgapur, Durgapur-713209*

Using the one-dimensional quantum hydrodynamic (QHD) model including the Poisson equation, Sagdeev's pseudopotential process is used to explore the propagation features of dust ion acoustic (DIA) solitary waves in an unmagnetized electron-positron-ion-dusty (e-p-i-d) quantum plasma. The asymptotic expansion is used to achieve the pseudopotential function for small values of the parameter H of quantum diffraction. QDIA solitary wave's existence domain is explored in terms of real Mach number boundaries. The numerical modeling results demonstrate that the dust grains can affect not just the amplitude and width, but also the soliton's domain of existence. The impacts of the quantum diffraction parameter on the soliton width are also addressed. It is also noted that the positron density may influence the propagation of the wave.

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I. INTRODUCTION

Study of nonlinear dust acoustic (DA) and dust ion acoustic (DIA) coherent structures, like, solitary waves, double layers have been studied in many scientific articles on dusty plasma[1–7]. Dusty plasmas are mainly constitute with electrons, ions and dust grains. Besides, the presence of positrons has also been reported in laboratories [8] and also in astrophysical bodies such as active galactic nuclei[9], the earlier universe[10], the pulsar magnetospheres[11], etc. Different kinds of linear and nonlinear wave constructions were explored in electron-positron (e-p) and electron-positron-ion (e-p-i) plasmas where plasma particles often obey Maxwellian velocity distribution. The strongly charged micron-sized dust particulates may occur in astrophysical objects along with electrons, positrons, and ions[12, 13]. The existence of electron-positron-ion-dust (e-p-i-d) plasma was noted in various space plasmas, i.e., hot spots on the "dust ring" in the galactic centre[14], inner areas of accretion disks close neutron stars and magnetars[15].

Haas *et al.*[16] studied the one-dimensional quantum hydrodynamic (QHD) model in the restriction of the charge carrier's small mass ratio and briefly discussed the impacts of the quantum diffraction parameter in both linear and non-linear quantity plasma environments. The quantum impacts in plasmas can occur in varying plasma systems. Quantum plasmas are analyzed primarily through two methods: Quantum Hydrodynamic (QHD) approach and the Quantum Kinetic approach. The mathematical formulation of the QHD system was provided long ago by Madelung[17]. A special force expression in the form of a Bohm potential gradient[18] appears in the momentum equation owing to the quantum tunneling effect. As the plasma particles follow the Fermi-Dirac distribution, the pressure term in the momentum equation is described in the Fermi pressure law. As the plasma particles obey the Fermi-Dirac distribution, the pressure term in the momentum equation is described by the Fermi pressure law, which includes the statistical quantum effects. Thus mathematical formulations for classical plasmas are appropriately altered by including these two quantitative features. The linear and nonlinear properties of quantum-ion-acoustic waves in e-p-i quantum plasmas have been studied by several authors[19–21]. However, dust impurities may exist in quantum plasmas. Using QHD model quantum ion acoustic wave has been explored in carbon nanostructures[22] and metallic nanowires[23].

II. BASIC EQUATIONS

We consider an unmagnetized four-component quantum dusty plasma (QDP) consisting of electrons, ions, positrons and negatively charged immovable dust particles. In order to explore QDIA waves in a QDP, the electrons and positrons are supposed to be inertialess and the phase velocity of the wave is assumed to be $v_{Fi} \ll \frac{\omega}{k} \ll v_{Fe}$ where v_{Fs} is the Fermi velocity of the electrons($s = e$) and ions($s = i$). Then the one dimensional quantum hydrodynamic (QHD) model for this system is governed by the following equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{e}{m_i}\frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$0 = e\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}(\sqrt{n_e})\right], \tag{3}$$

*Communication address: Email:gban.iitkgp@gmail.com

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$$0=-e\frac{\partial\phi}{\partial x}-\frac{1}{n_p}\frac{\partial p_p}{\partial x}+\frac{\hbar^2}{2m_p}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_p}}\frac{\partial^2}{\partial x^2}(\sqrt{n_p})\right], \quad (4)$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi e(\ n_e+Z_{d0}n_{d0}-n_i-n_p), \quad (5)$$

where $n_s,\ v_s,\ m_s$ are the number density, fluid velocity and mass of the species electron ($s=e$) and ion ($s=i$) respectively, ϕ is the electrostatic potential, n_{d0} is the equilibrium dust number density, Z_{d0} is the number of electrons residing on the dust grains, $\hbar$ is the Planck's constant and $-e$ (e) is the electron (ion and positron) charge. Here $m_e=m_p$ and the electrons ($s=e$) and positrons ($s=p$) are assumed to follow the one dimensional zero-temperature Fermi gas pressure law[24]

$$p_e=\frac{m_e v_{Fe}^2}{3n_{e0}^2}n_e^3, \quad (6)$$

where the Fermi electrons ($s=e$) and positrons ($s=p$) velocity is given by $v_{Fs}=\sqrt{2K_BT_{Fs}/m_s}$, K_B is the Boltzmann constant and T_{Fs} is the Fermi temperature. The charge neutrality condition at equilibrium is given by

$$n_{i0}+n_{p0}=n_{e0}+Z_{d0}n_{d0}. \quad (7)$$

Now, eqs. (1)-(5) can be written in the normalized form as follows:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_iv_i)=0, \quad (8)$$

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}=-\frac{\partial\phi}{\partial x}, \quad (9)$$

$$0=\frac{\partial\phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2\delta}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}(\sqrt{n_e})\right], \quad (10)$$

$$0=-\frac{\partial\phi}{\partial x}-\sigma n_p\frac{\partial n_p}{\partial x}+\frac{H^2}{2\delta}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_p}}\frac{\partial^2}{\partial x^2}(\sqrt{n_p})\right], \quad (11)$$

$$\frac{\partial^2\phi}{\partial x^2}=\mu_e\ n_e-\mu_pn_p-n_i+\mu_d, \quad (12)$$

where the wave potential ϕ, the ion fluid velocity v_i and the number densities n_s are normalized by $\frac{2K_BT_{Fe}}{e}$, quantum ion acoustic speed $C_0=\left(\frac{2K_BT_{Fe}}{m_i}\right)^{\frac{1}{2}}$ and the unperturbed number densities n_{s0} ($s=e,i$), respectively. The space and time coordinates are normalized by the ion Fermi wave length in quantum plasma, $\lambda=\left(\frac{2K_BT_{Fe}}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$ and ion plasma period $\omega_{pi}^{-1}=\left(\frac{m_i}{4\pi n_{i0}e^2}\right)^{\frac{1}{2}}$, respectively. Here the dust density parameter $d=\frac{Z_{d0}n_{d0}}{n_{e0}}$, $\delta=\frac{n_{i0}}{n_{e0}}$, $p=\frac{n_{p0}}{n_{e0}}$, $\mu_e=\frac{1}{1-p+d}$, $\mu_p=\frac{p}{1-p+d}$, $\mu_d=\frac{d}{1-p+d}$ electron plasma period $\omega_{pe}=\left(\frac{4\pi n_{e0}e^2}{m_e}\right)^{\frac{1}{2}}$ and the nondimensional quantum parameter H is defined as $H=\frac{\hbar\omega_{pe}}{2K_BT_{Fe}}$. The charge neutrality condition (7) implies $\delta=1-p+d$. The Fermi temperature T_{Fs} ($s=p,e$) is defined as $K_BT_{Fs}=\frac{\hbar^2(3\pi)^{\frac{2}{3}}n_s^{\frac{2}{3}}}{2m}$, $\sigma=\frac{T_{Fp}}{T_{Fe}}=p^{2/3}$.

III. OBTAINING SAGDEEV POTENTIAL

For obtaining the traveling wave solutions of the eqs. (8)-(12) that are stationary in a frame moving with a velocity M, we assume that all the dependent variables depend on $\xi=x-Mt$, M being the Mach number normalized to the quantum ion acoustic speed C_s. Then eqs. (8) and (9) reduce to

$$n_i=\frac{M}{M-v_i}, \quad (13)$$

$$(v_i-M)^2=M^2-2\phi, \quad (14)$$

where we have imposed the boundary conditions $n_i\to 1,\ v_i\to 0$ and $\phi\to 0$ as $\xi\to\ \pm\infty$. Then eqs. (13) and (14) imply

$$n_i=\frac{1}{\sqrt{1-\frac{2\phi}{M^2}}}, \quad (15)$$

Eqs. (10) and (11) reduce to

$$n_e^2=1+2\phi+\frac{H^2}{\delta}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial\xi^2}(\sqrt{n_e})\right], \quad (16)$$

$$n_p^2=1-\frac{2\phi}{\sigma}+\frac{H^2}{\delta\sigma}\left[\frac{1}{\sqrt{n_p}}\frac{\partial^2}{\partial\xi^2}(\sqrt{n_p})\right], \quad (17)$$

where we have imposed the boundary conditions, $\phi\to 0,\ n_e\to 1,\ n_p\to 1,\ \frac{\partial^2}{\partial\xi^2}(\sqrt{n_e})\to 0$ and $\frac{\partial^2}{\partial\xi^2}(\sqrt{n_p})\to 0$ as $\xi\to\ \pm\infty$. By observing the nature of the terms of eq. (16), it is expected that n_e must be a function of ϕ. So for small value of H, we suppose

$$n_e^2=f_0(\phi)+H^2f_1(\phi)+O(H^4), \quad (18)$$

where $f_0(\phi)$ is the electron number density at $H=0$ and $O(H^4)$ term is neglected. Using eq. (18) in eq. (16) and comparing the coefficients upto the order of H^2 we get

$$f_0(\phi)\ =\ 1+2\phi, \quad (19)$$

$$f_1(\phi)\ =\ \frac{1}{\delta}(1+2\phi)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}(1+2\phi)^{\frac{1}{4}}, \quad (20)$$

Then, eq. (18) implies

$$n_e^2 = 1 + 2\phi + \frac{H^2}{\delta}\left[(1+2\phi)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}(1+2\phi)^{\frac{1}{4}}\right]. \tag{21}$$

In a similar way the positron density expression is obtained as

$$n_p^2 = 1 - \frac{2\phi}{\sigma} + \frac{H^2}{\delta\sigma}\left[\left(1-\frac{2\phi}{\sigma}\right)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}\left(1-\frac{2\phi}{\sigma}\right)^{\frac{1}{4}}\right]. \tag{22}$$

Eqs. (21) and (22) express the electron and positron densities as a function of the electrostatic potential which are derived on the basis of the semiclassical limit where H is taken to be small[19, 25]. Now substituting the density expressions from eqs. (15), (21) and (22) in the Poisson equation, we obtain

$$\frac{d^2\phi}{d\xi^2} = \mu_e\sqrt{1+2\phi+\frac{H^2}{\delta}\left[(1+2\phi)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}(1+2\phi)^{\frac{1}{4}}\right]}$$
$$-\mu_p\sqrt{1-\frac{2\phi}{\sigma}+\frac{H^2}{\delta\sigma}\left[\left(1-\frac{2\phi}{\sigma}\right)^{-\frac{1}{4}}\frac{\partial^2}{\partial\xi^2}\left(1-\frac{2\phi}{\sigma}\right)^{\frac{1}{4}}\right]} - \frac{1}{\sqrt{1-\frac{2\phi}{M^2}}} + \mu_d. \tag{23}$$

To obtain the Sagdeev potential function $V(\phi)$, which satisfies the energy integral form

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{24}$$

we suppose

$$V'(\phi) = -\frac{d^2\phi}{d\xi^2} \tag{25}$$

and so

$$V(\phi) = -\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2. \tag{26}$$

Here the charge neutrality condition gives

$$V(0) = V'(0) = 0. \tag{27}$$

Making use of eqs. (26) and (25), eq. (23) reduces to

$$V'(\phi) = -\mu_e\sqrt{(1+2\phi)+\frac{H^2}{\delta}\left[\frac{3\,V(\phi)}{2(1+2\phi)^2}-\frac{V'(\phi)}{2(1+2\phi)}\right]}$$
$$+\mu_p\sqrt{\left(1-\frac{2\phi}{\sigma}\right)+\frac{H^2}{\delta}\left[\frac{3\,V(\phi)}{2\sigma^3\left(1-\frac{2\phi}{\sigma}\right)^2}+\frac{V'(\phi)}{2\sigma^2\left(1-\frac{2\phi}{\sigma}\right)}\right]} + \frac{1}{\sqrt{1-\frac{2\phi}{M^2}}} - \mu_d. \tag{28}$$

Here, first neglecting the quantum diffraction effect ($i.e.$ $H=0$) from the above eq. (28), we get

$$V_0{}'(\phi) = -\mu_e\sqrt{(1+2\phi)} + \mu_p\sqrt{\left(1-\frac{2\phi}{\sigma}\right)} + \frac{1}{\sqrt{1-\frac{2\phi}{M^2}}} - \mu_d, \tag{29}$$

where $V_0'(\phi)$ denotes $V'(\phi)$ at $H = 0$. Integrating eq. (29) and using the condition (27), we get

$$V_0(\phi) = \frac{1}{3}\mu_e\left[1-(1+2\phi)^{3/2}\right] + \frac{1}{3}\mu_p\sigma\left[1-\left(1-\frac{2\phi}{\sigma}\right)^{3/2}\right] + M^2\left[1-\sqrt{1-\frac{2\phi}{M^2}}\right] - \phi\mu_d. \tag{30}$$

In order to obtain $V(\phi)$, on the basis of the semiclassical limit where H is taken to be small so that the terms of $O(H^4)$ can be neglected, we suppose that

$$V(\phi) = V_0(\phi) + H^2 V_1(\phi) + O(H^4). \tag{31}$$

Then eq. (28) implies

$$V'(\phi) = -\mu_e\sqrt{(1+2\phi) + \frac{H^2}{\delta}\left[\frac{3\ V_0(\phi)}{2(1+2\phi)^2} - \frac{V_0'(\phi)}{2(1+2\phi)}\right]}$$
$$+\mu_p\sqrt{\left(1-\frac{2\phi}{\sigma}\right) + \frac{H^2}{\delta}\left[\frac{3\ V_0(\phi)}{2\sigma^3\left(1-\frac{2\phi}{\sigma}\right)^2} + \frac{V_0'(\phi)}{2\sigma^2\left(1-\frac{2\phi}{\sigma}\right)}\right]} + \frac{1}{\sqrt{1-\frac{2\phi}{M^2}}} - \mu_d. \tag{32}$$

The above equation (32) is valid only in the semiclassical limit of H where the accuracy is upto the order of H^2. Then $V(\phi)$ is obtained from eq. (32) by numerical integration. It is observed from the density expressions (15), (21) and (22) that in order to prevent wave braking there are limitations on ϕ that $-\frac{1}{2} < \phi < \min\left\{\frac{\sigma}{2}, \frac{M^2}{2}\right\}$.

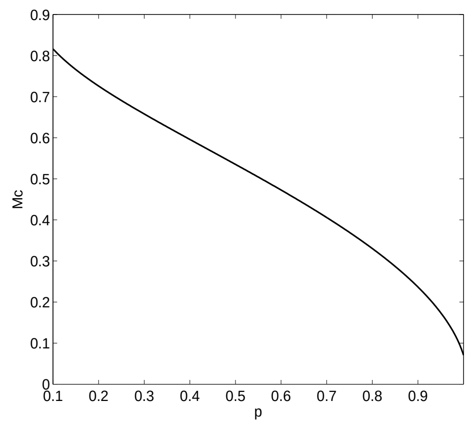

FIG. 1: *Variation of critical Mach number M_c against p. Here, $H = 0.087$, $d = 0.2$.*

To study the existence of solitary waves, we analyze the Sagdeev potential $V(\phi)$ which satisfy the following

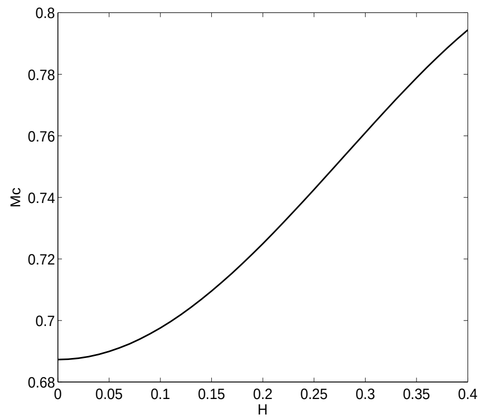

FIG. 2: *Variation of critical Mach number M_c against H. Here, $p = 0.25$, $d = 0.2$.*

Sagdeev's conditions[26]

(i) $V''(\phi) < 0$ at $\phi = 0$, so that the fixed point at the origin is unstable.

(ii) $\exists$ a nonzero ϕ_m, the maximum (or minimum) value of ϕ, at which $V(\phi) = 0$.

(iii) $V(\phi) < 0$, for $0 < |\phi| < |\phi_m|$.

Condition (i) gives the lower limit of M for existence of solitary waves as $M > M_c$ where

$$M_c = \sqrt{\frac{1 - \frac{H^2}{4\sqrt{1-H^2}} + \frac{H^2\mu_p}{4\sigma^2\mu_e\sqrt{1+\frac{H^2}{\sigma^2}}}}{\frac{\mu_e}{2\sqrt{1-H^2}}\left\{2 + 2H^2 + \frac{H^2}{2\mu_e}\left(\frac{\mu_e(1-\sigma)-1}{\sigma}\right)\right\} - \frac{p-d}{2\sqrt{1+\frac{H^2}{\sigma^2}}}\left\{-\frac{2}{\sigma} + \frac{2H^2}{\sigma^3} - \frac{H^2}{2\mu_e\sigma^2}\left(\frac{\mu_e(1-\sigma)-1}{\sigma}\right)\right\}}}. \quad (33)$$

Here for $H = 0$, we have $M_c = \sqrt{\frac{\sigma}{\mu_e(1+\sigma)-1}}$ which is similar as obtained by Popel *et al.*[27]. In absence of positron and dust, *i.e.* if $p \to 0$ and $d \to 0$, $M_c \to 1$ which is obtained same as that of simple electron-ion plasmas.

IV. NUMERICAL RESULTS AND DISCUSSION

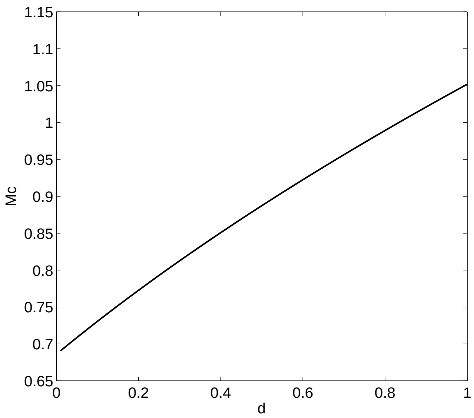

FIG. 3: *Variation of critical Mach number M_c against d. Here, $p = 0.25$, $H = 0.087$.*

The numerical findings have been addressed in this section. The values of the parameters are taken as[20, 25, 28, 29]: $n_{e0} \sim 5 \times 10^{29}$, $n_{p0} \sim 0.2 n_{e0}$ $T_{Fe} \sim 10^8 K$, $Z_{d0} \sim 10^3$. It is noted from eq. (33) that the critical Mach number depends upon the quantum diffraction parameter H, the positron-ion density ratio p and dust-ion density ration d. The variation of the critical Mach number M_c with p, H and d is plotted in Fig. 1, Fig. 2 and Fig. 3, respectively. It is found that the critical Mach number decreases with the increase in positron-electron density ratio, whereas it increases due to the increase in H and d. It should be pointed out here that, since the value of M depends on a particular normalization, some care should be given to interpret the outcomes physically. So the true Mach number in the system is described by the ratio M/Mc. From this proportion, the reference velocity of C_0 used in the normalization of M disappears[30, 31]. Thus the existence condition for solitary waves $M > M_c$ implies that the true mach number $M/M_c > 1$. In Fig. 4, the Sagdeev potential $V(\phi)$ is plotted for different values of p keeping other parameter values fixed. It is observed that as positron density increases the amplitude of the solitary wave decreases. On the other hand, it has been observed from Fig. 5, where

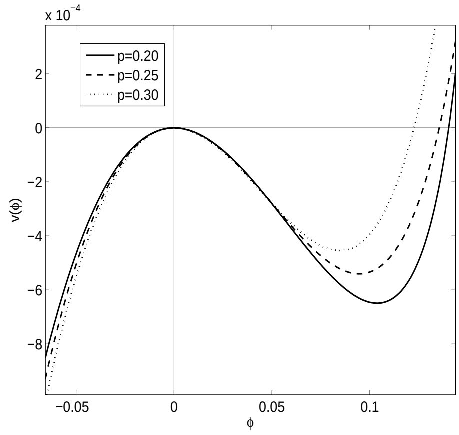

FIG. 4: *Plot of $V(\phi)$ for different values of p. Here, $H = 0.087$, $d = 0.2$.*

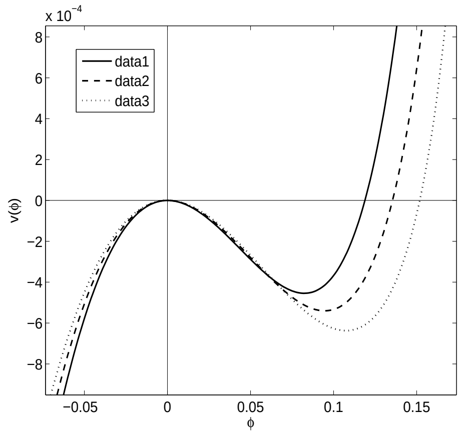

FIG. 5: *Plot of $V(\phi)$ for different values of d. Here, $H = 0.087$, $p = 0.25$.*

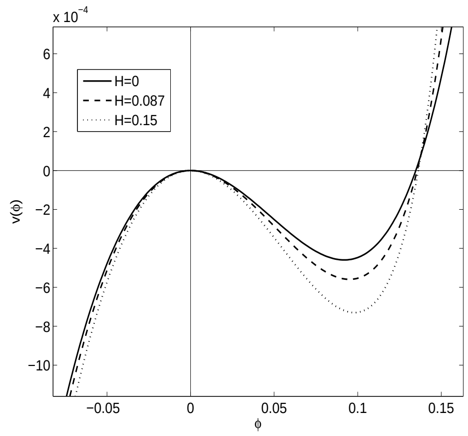

FIG. 6: *Plot of $V(\phi)$ for different values of H. Here,* $p = 0.25$, $d = 0.2$.

$V(\phi)$s are plotted for different values of d, that the amplitude of the solitary waves increases with increasing dust density, keeping other parameter values fixed. In Fig. 6, $V(\phi)$s are plotted for different values of H. Here we found that in the given range of quantum diffraction parameter H, very small change in solitary wave amplitude has been observed.

V. CONCLUSIONS

In this paper, the propagation properties of the QDIA solitary wave have been explored by using Sagdeev's pseudopotential technique along with the Poisson equation in an unmagnetized e-p-i-d plasma. Here, the asymptotic expansion method, for small values of the quantum diffraction parameter H, have been used to obtain the Sagdeev potential. Existence of arbitary amplitude solitary wave have been predicted, theoretically. The amplitude of the electrostatic potential framework is also improved owing to the presence of dust particles and positrons in quantum plasmas. Small impacts of the quantum diffraction parameter on the soliton amplitude are also noted.

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